TOPIC 1 OVERVIEW

Introduction to Exponential Functions

How are the key concepts of *Introduction to Exponential Functions* organized?

In *Introduction to Exponential Functions*, students learn and apply properties of integer and rational exponents to simplify numeric and algebraic expressions. First, students review their knowledge of powers and broaden the set of numbers they use as bases to include negative integers. Students differentiate between cases where there is a negative in the base and cases where the negative indicates the opposite of the power. They then explore products of powers, quotients of powers, and powers of powers to develop rules for each operation. Next, students search for patterns to determine rules for all integer exponents, including 0 and negative integers. They simplify numeric and algebraic expressions that contain multiple exponents and use the exponent rules to justify their steps.

Integer & rational exponents

products & quotients of powers.

Students then build upon their previous understanding of sequences and common ratio to recognize that some geometric sequences are exponential functions and others are not. They can already write an explicit and recursive formula given a geometric sequence, but in this topic, they rewrite explicit formulas in function form for exponential functions. Students connect the common ratio of

MATH Repress.

Math Representation

Jasmine used the idea of the constant multiplier to estimate the value of $f\left(\frac{1}{2}\right)$ for the function $f(x) = 2^x$.

Jasmine

I know the constant multiplier for an interval of 1 is 2. I want to split each interval of 1 into two equal parts, which means I need two equal multipliers.

So, $r^2 = 2$.

x	0	$\frac{1}{2}$	1
$f(x)$	2^0	$2^{\frac{1}{2}}$	2^1
	1		2

$1 \cdot r \cdot r = 2$

Multiply by 2.

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a geometric sequence with the base of the power in an exponential function of the form $f(x) = ab^x$. They prove algebraically that there is always a constant ratio between consecutive output values of an exponential function.

Students then analyze the structure of square root expressions using squares and their side lengths. Students decompose the side lengths of both perfect and non-perfect squares to reveal the properties of radicals and extracting perfect squares. Students use the properties of radicals to multiply, divide, and simplify square root expressions.

Next, students examine the structure of exponential functions. The domain of a geometric sequence is constrained to whole number inputs, whereas the domain of an exponential function is the set of real numbers. Given the function $f(x) = 2^x$, students explore the constant ratio over the intervals $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. Through this exploration, they conclude that $2^{\frac{1}{2}} = \sqrt{2}$ and $2^{\frac{1}{3}} = \sqrt[3]{2}$. By graphing these values, the misconception that $f\left(\frac{1}{2}\right)$ is halfway between $f(0)$ and $f(1)$ is addressed. Students generalize the rule for rational exponents and practice rewriting expressions with rational exponents as radicals and vice versa. In addition to rational input values, students also consider the value of an exponential function when the input is increasingly negative. They define *horizontal asymptote* and identify instances on graphs of exponentials.

What is the entry point for students?

Students have learned to write and evaluate numeric and algebraic expressions with whole number exponents in Grade 6. They have also learned to estimate the square roots of non-perfect squares. In this topic, students expand on this knowledge as they learn and apply properties of exponents to simplify numeric and algebraic expressions.

Math Representation

The recursive formula to determine the *n*th term of a geometric sequence is:

$$g_n = g_{n-1} \cdot r$$

(g_n: *n*th term; g_{n-1}: previous term; r: common ratio)

The explicit formula to determine the *n*th term of a geometric sequence is:

$$g_n = g_1 \cdot r^{n-1}$$

(g_n: *n*th term; g_1: 1st term; r: common ratio; $n-1$: previous term number)

Previously in this course in the topic *Sequences*, students were introduced to geometric sequences. Students can already write a recursive and an explicit formula for a given geometric sequence. In this topic, they learn to write geometric sequences as exponential functions, where possible, and to compare the structures of the exponential functions.

Why is *Introduction to Exponential Functions* important?

This topic highlights the uniqueness of exponential functions—relationships that constantly grow or decay—but frames the learning within a larger set of functions. Throughout this topic, students apply what they know about the key characteristics of functions (e.g., intercepts, intervals of increasing or decreasing, and domain and range) to include exponential functions.

> **Math Representation**
>
> Given the parent function $y = f(x)$, you have learned how to identify the effects and graph a function written in the transformation form $g(x) = a \cdot f(b(x - c)) + d$.
>
> **For quadratic functions specifically, you will also see them written in the form $f(x) = a(x - h)^2 + k$, where $a \neq 0$. This is vertex form.**
>
> The variable h represents the x-coordinate of the vertex.
>
> The variable k represents the y-coordinate of the vertex.
>
> You can determine whether the parabola opens up or down and the coordinates of the vertex from the vertex form of a quadratic function.

This deepens the knowledge students have built about linear functions in **Exploring Constant Change** and prepares them for the work they will be doing with quadratic functions, both in **Maximizing and Minimizing** and with more complex functions in future courses.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Introduction to Exponential Functions* when they can:

- Use the rules of integer exponents to simplify numeric and algebraic expressions.
- Justify the exponent rules, including product of powers, quotient of powers, power of a power, and zero and negative exponent rules.
- Write an equation in function notation for an exponential function represented as a table, a graph, a set of ordered pairs, or a scenario.
- Evaluate an exponential function for any unknown value.
- Use the equation, graph, or scenario to identify key characteristics of an exponential function, including the y-intercept, the horizontal asymptote, the x-intercept, and intervals of increase or decrease.
- Describe what the key characteristics of a function indicate about the problem scenario.
- Create a graph, including all of the key characteristics, that matches a given scenario.
- Explain that the parent function for an exponential function is $f(x) = ab^x$, where $a = 1$ and $b > 0$.

NEW KEY TERMS

- power [potencia]
- base [base]
- exponent [exponente]
- horizontal asymptote [asíntota horizontal]
- perfect square
- square root
- radical [radical]
- radicand [radicando]
- product property of radicals [propiedad del producto de radicales]
- extracting perfect squares
- index [índice]
- quotient property of radicals [propiedad del cociente de radicales]

- Demonstrate that an exponential function has a constant ratio over equal intervals.
- Identify an exponential function from a table of values, an equation, a graph, or a scenario.
- Use the properties of radicals to simplify square root expressions.
- Use the rules of exponents to rewrite numeric and algebraic expressions with rational exponents.
- Apply the rule $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ to rewrite expressions.

How do the activities in *Introduction to Exponential Functions* promote student expertise in the TEKS mathematical process standards?

Each topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Students examine sequences of operations or numbers to derive exponent rules; once they recognize a shortcut, they can write their rule (A.1F). Students work to develop fluency with the rules, justifying steps in the simplification process, and critiquing others' work (A.1B).

Students spend extensive time in this topic examining the structure of exponential functions, comparing and contrasting them with linear functions (A.1F). They use tables, equations, and graphs to compare functions but also use these same tools to solve problems and make predictions (A.1D). Students use reasoning to identify functions representing exponential growth or exponential decay (A.1D).

How can you use cognates to support EB students?

Supports for EB Learners "Bilingual Bingo"

Cognates are provided for new key terms when applicable. Play games with students that build connections between languages, such as Bilingual Bingo, where you call out math terms or word problems in one language, and students must locate or solve them in the other language. Games like this can also help reinforce students' familiarity with cognate patterns through repeated exposure and practice.

MODULE 4, TOPIC 1 PACING GUIDE

165-Day Pacing

4 Investigating Growth and Decay

TOPIC 1: Introduction to Exponential Functions

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.B, 1.E, 1.G, 2.A, 2.D, 2.H, 2.I, 3.B, 3.D, 3.F, 3.H, 4.C, 5.B, 5.C, 5.D, 5.E

Topic Pacing: 15 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
1	Properties of Powers with Integer Exponents	The terms *power, base of a power*, and *exponent of a power* are defined. Students write and evaluate expressions with positive integer exponents. They begin with a context using the power with a base of 2. Students then investigate positive and negative integer bases where the negative sign may or may not be raised to a power depending on the placement of parentheses. Some expressions also contain variables. **Materials Needed:** Calculator	**A.11B**	3
2	Analyzing Properties of Powers	Students use the properties of powers to justify each step when rewriting expressions with exponents. They solve additional practice problems and examine student work. Students demonstrate their understanding of the properties of powers by creating graphic organizers. **Materials Needed:** Scissors	**A.11B**	2
3	Geometric Sequences and Exponential Functions	In Module 2, Topic 1, *Lesson 3: Making Connections Between Arithmetic Sequences and Linear Functions*, students rewrote the explicit form of an arithmetic sequence as a linear function and proved algebraically that the common difference in a linear function is the slope of the line. This lesson follows a similar process, where they revisit geometric sequences as a launch to exponential functions. Students know that all geometric sequences are functions, and through investigation, they learn that some geometric sequences are exponential functions, while others are not. They identify the fact that the constant ratio must be positive for a geometric sequence to be an exponential function. Through a context, student work, and a Worked Example, students use properties of exponents to rewrite the explicit form of a geometric sequence as a function in the form $f(x) = ab^x$ and make connections between the two forms. Students then explore a situation modeled by an exponential function. They are guided to demonstrate that the ratio between consecutive output values of any exponential function is constant and is represented by the variable b in the function form $f(x) = ab^x$, and the y-intercept is represented by the ordered pair $(0, a)$. Students then solve a problem in context that is represented by a decreasing exponential function. **Materials Needed:** Problem-Solving Model Graphic Organizer	A.9B **A.9C** **A.9D** **A.11B** A.12D **A.12C**	2

READINESS TEKS

*Bold TEKS = Readiness Standard

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Lesson	Lesson Title	Highlights	TEKS*	Pacing
4	**Rewriting Square Roots**	Students analyze perfect square models and models of non-perfect squares in terms of their areas and side lengths. Students understand that the areas of the squares represent radicands, and the side lengths represent the square roots of those radicands. They write the side lengths of square models both as single square roots and as multiplication expressions. This allows students to model the simplification of square roots and make observations leading to the properties of radicals. Students then use the properties to multiply, divide, and simplify square roots by extracting perfect squares and by using prime factorization. Students discuss strategies for efficiently simplifying radicals. **Materials Needed:** None	A.11A	1
5	**Rational Exponents and Graphs of Exponential Functions**	Students explore a context modeled by an exponential function, first with output values that are between two integers, then with output values that are rational exponents. This helps students make sense of the fact that $\sqrt{a} = a^{\frac{1}{2}}$. They then expand upon this idea to learn that the properties of powers apply to expressions with rational exponents, rewrite expressions with rational exponents as radicals, and then connect the two concepts to perform and justify operations involving radicals. Students explore the effects of a negative exponent, learn the meaning of a *horizontal asymptote*, and analyze this idea of end behavior on several graphs. **Materials Needed:** None	A.9A A.9B **A.9C** **A.9D** A.11A **A.11B** A.12B	3
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				3

READINESS TEKS

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MODULE 4, TOPIC 1 PACING GUIDE

~~165-Day Pacing~~

150 Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: **A.11B** **LESSON 1** Properties of Powers with Integer Exponents GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 1** continued ACTIVITY 3 ACTIVITY 4	**LESSON 1** continued ACTIVITY 5 ACTIVITY 6 TALK THE TALK	**LEARNING INDIVIDUALLY** Skills Practice *This is a suggested placement. Move based on student data and individual needs.*	TEKS: **A.11B** **LESSON 2** Analyzing Properties of Powers GETTING STARTED ACTIVITY 1
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
LESSON 2 continued ACTIVITY 2 TALK THE TALK	TEKS: A.9B, **A.9C, A.9D, A.11B,** A.12D, A.12C **LESSON 3** Geometric Sequences and Exponential Functions GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 3** continued ACTIVITY 3 ACTIVITY 4 TALK THE TALK	**LEARNING INDIVIDUALLY** Skills Practice *This is a suggested placement. Move based on student data and individual needs.*	TEKS A.11A **LESSON 4** Rewriting Square Roots GETTING STARTED ACTIVITY 1 ACTIVITY 2 TALK THE TALK
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**
TEKS: A.9A, A.9B, **A.9C, A.9D,** A.11A, **A.11B,** A.12B **LESSON 5** Rational Exponents and Graphs of Exponential Functions GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 5** continued ACTIVITY 3 ACTIVITY 4	**LESSON 5** continued ACTIVITY 5 TALK THE TALK	**LEARNING INDIVIDUALLY** Skills Practice *This is a suggested placement. Move based on student data and individual needs.*	**END OF TOPIC ASSESSMENT**

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How can you incorporate Skills Practice with students?

There are three Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets can be assigned to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice sets of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.